Financial Conditions and the Risks to Economic Growth in the United States Since 1875*

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Abstract

We explore the historical relationship between financial conditions and economic growth for quarterly U.S. data from 1875 to 2017 with a flexible empirical copula modelling methodology. We compare specifications with both linear and non-linear dependence, and with both Gaussian and non-Gaussian marginal distributions. We find strong support for models that are both non-Gaussian and non-linear, with considerable heterogeneity across sub-samples. Ignoring the contribution of financial conditions typically understates the conditional downside risks to economic growth in crises. For example, accounting for financial conditions more than doubles the probability of negative growth in the year following the 1929 stock market crash.

Keywords: Financial Conditions; Growth at risk; Vulnerable Growth.

JEL codes: C14; C32; C53; E37; E44; N10; N20

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1 Introduction

Some important recent empirical macroeconomic research highlights the impact of financial conditions on the risks to economic growth. Papers by, among others, Adrian, Boyarchenko and Giannone (2019a, ABG) and Adrian et al (2018) argue that economic growth has a non-Gaussian distribution. Their research quantifies the vulnerability of economic growth using quantile regression methods and provides the basis for the IMF's bilateral surveillance tool, "growth at risk". In this paper, we use a copula modelling strategy with sufficient flexibility to encompass both non-Gaussianity and non-linearity to take a broad historical perspective of the relationship between financial conditions and real economic growth using quarterly U.S. data from 1875 to 2017.

Looking across sub-samples of historical U.S. data and across a variety of measures for both financial conditions and economic growth, we find financial conditions matter for economic growth and strong statistical support for both non-Gaussianity and non-linearity. Furthermore, restricting attention to linear dependence limits the predictive content from financial conditions, regardless of whether the fitted marginal distributions are Gaussian. These results hold across all our sub-samples, even though there is considerable heterogeneity across historical eras.

We consider the impact of financial conditions on the risks to economic growth based on our preferred non-Gaussian and non-linear copula specification. We find that ignoring the contribution of financial conditions typically understates the conditional risks to economic growth. For example, including financial conditions more than doubles the risk of negative economic growth from 31 percent to 79 percent for the year following the 1929 stock market crash.

In terms of our modelling methodology, we build on recent time series research adopting copula methods to separate margins from dependence. Following, among others, Smith (2015), Smith and Vahey (2016), Loaiza-Maya and Smith (2019) and Karagedikli, Vahey and Wakerly (2019), we fit non-parametric marginal distributions based on the Empirical Cumulative Distribution Function (ECDF) for each macroeconomic variable. In this manner, we provide scope for non-Gaussianity in the macroeconomic variables, while modelling both cross-sectional and serial dependence in the copula density.

Unlike these previous macroeconomic copula studies, we adopt an empirical copula methodology, fitting the dependence parameters by deploying a kernel density estimator, with adjustments for boundaries, using non-parametric estimation on pseudo data. In contrast to our approach, direct kernel density estimation based on the observed time series would combine any non-Gaussianity and non-linearity; DiNardo and Tobias (2001) and the density impulse responses of Adrian, Boyarchenko and Giannone (2019b) provide examples of that approach.

In addition to ABG (2019a), Chavleishvili and Manganelli (2019), Ferrara, Mogliani, and Sahuc (2019) and Loria, Matthes and Zhang (2019) all adopt (modified) quantile regression methods to study the importance of non-Gaussianity for economic growth with modern era data. Bernard and Czado (2015) discuss robustness issues for dependence estimates delivered by quantile regression methods in the presence of non-Gaussianity. Amengual, Sentana and Tian (2019) propose robust methods to compute linear dependence with copulas. (We adapt their approach to estimate our non-Gaussian and linear specifications.) Carriero, Clark and Marcellino (2019) note that the modern era U.S. data contain relatively few pertinent observations to assess tail behaviour for output growth. Historical data give further scope to study predictability for economic growth events.

The remainder of this paper is structured as follows. In Section 2, we set out our macroeconomic copula modelling methodology, describing how the approach has sufficient flexibility to separate non-Gaussian and non-linear features for the variables explored by ABG (2019a). In Section 3, we summarize the U.S. historical data. We consider the same National Financial Conditions Index (NFCI) for modern data as ABG (2019a), a factor model based index of over 100 indicators of financial conditions, in addition to our other measures of financial conditions. In Section 4, we present our prediction results and for the risks to economic growth. In the final section, we conclude by briefly discussing a future avenue for research utilizing non-Gaussian and non-linear models of economic growth.

2 Macroeconomic Copula Modelling

In this section, first we describe how the copula modelling methodology separates margins from dependence and then we give a detailed account of our approach to fitting the empirical copula models to macroeconomic data. We conclude this section by describing prediction and evaluation.

2.1 Separation of Margins and Dependence

To show how copulas help distinguish between non-Gaussian and non-linear features in general, we exploit Skars' Theorem (Nelsen, 2006) to decompose a joint distribution into univariate margins and dependence.

Suppose we consider a multivariate joint distribution for S variables stacked in $\boldsymbol{Z}$, with a (row) vector of time series observations for each variable. Sklar's Theorem indicates that a unique copula exists, C, under certain regularity conditions, which separates the joint distribution into the univariate margins and the dependence structure. We express the joint distribution function for $\boldsymbol{Z}$ using the copula function, C:

$$F(\boldsymbol{z}) = C(\boldsymbol{u}) \tag{1}$$

where $\boldsymbol{z} = (\boldsymbol{z}_1', \ldots, \boldsymbol{z}_S')'$ and $\boldsymbol{u} = (\boldsymbol{u}_1', \ldots, \boldsymbol{u}_S')'$. The joint distribution on the right hand side, C, is defined on the S-dimensional unit cube, $[0,1]^S$. The S variables in $\boldsymbol{u}$ are often referred to as "copula data". Denoting the Cumulative Distribution Function, $F(\cdot)$, we can define the copula data as $\boldsymbol{u}_s = F_s(\boldsymbol{z}_s)$ so that they are individually uniformly distributed for each margin.

We differentiate the distribution function, equation (1), to give the probability density of $\boldsymbol{Z}$ as the product of the copula density and the S marginal densities:

$$f(\boldsymbol{z}) = c(\boldsymbol{u}) \prod_{s=1}^{S} f_s(\boldsymbol{z}_s) \tag{2}$$

where $f_s(\boldsymbol{z}_s)$ is the marginal density of $\boldsymbol{z}_s$ and $c(\boldsymbol{u}) = \frac{\partial}{\partial \boldsymbol{u}} C(\mathbf{u})$ is the copula density.

With the copula density representing the dependence on the unit cube, and non-linearity being a property of the dependence, non-Gaussianity is captured separately by the margins.

2.2 Fitting Empirical Copulas to Macroeconomic Variables

Each of the S macroeconomic variables refers to a time series of length T with corresponding copula data $\boldsymbol{u}_s = (u_{s,1}, \ldots, u_{s,T})'$. We assume that the time series variables are stationary; see Smith (2015) for a discussion of stationarity and copula modelling.

The Probability Integral Transforms provide computationally convenient copula data derived from the Empirical Cumulative Distribution Function (ECDF) for each variable. Previous applications working with ECDF marginal distributions on modern macroeconomic data include Smith and Vahey (2016), Loaiza-Maya and Smith (2019) and Karagedikli, Vahey and Wakerly (2019). Unlike those papers, which fit parametric copula densities, we adopt a flexible route to fitting dependence by adapting empirical copula methods. Empirical copulas have both non-parametric marginals and a non-parametric copula density; see Deheuvals (1979), Deheuvals (1981) and, for a recent example, Velásquez-Giraldo et al (2018).

Empirical copula studies typically utilize rank-transformed counterparts to the copula data, sometimes referred to as "pseudo data", to mitigate the danger of misspecification in fitting the marginal distributions. For each macroeconomic variable, indexed by s, we define the pseudo data as $V_{s,t} = R_{s,t}/(T+1)$, where for each variable, R denotes the rank of the t^{th} observation relative to its own history, for $t = 1, \ldots, T$. The $T+1$ denominator avoids boundary issues.

We consider a similar model space to ABG (2019a) with three macroeconomic variables, so that $S = 3$. Namely, output growth and lagged financial conditions, with lagged output growth also included as a predetermined conditioning variable. We fit the copula density non-parametrically to the three (rank-transformed) pseudo variables to capture dependence. With the dependence estimates conditional on the fitted marginals, this limited information estimation strategy is sometimes known as "inference for margins"; see, Joe (2006).

We fit the multivariate pseudo time series using a kernel density estimator (KDE). Among others, Silverman (1986), Scott (1992), DiNardo and Tobias (2001), Li and Racine (2006) and Adrian, Boyarchenko and Giannone (2019b) adopt KDE methods but not within a copula framework. We utilise a Beta kernel to fit the multivariate distribution on the S-dimensional unit cube, reducing scope for boundary bias; see the discussions in Chen (1999), Charpentier, Fermanian and Scaillet (2007) and Karra (2018). The fitted copula density is:

$$\frac{1}{T}\sum_{t=1}^{T}\prod_{s=1}^{3} K(V_{s,t}, \alpha, \beta) \tag{3}$$

where K denotes the Beta density with parameters $\alpha = \frac{v}{h} + 1$ and $\beta = \frac{1-v}{h} + 1$ and h denotes bandwidth. Following Adrian, Boyarchenko and Giannone (2019b), we select the bandwidth for our

KDE based on out of sample performance for the modern era. We describe this process in detail in the online appendix. We utilize the same bandwidth value for the historical data prior to 1970.

Mindful of the potential complexity of economic growth in financial crises, the capacity of our empirical copula methodology to handle both non-Gaussianity and non-linearity is appealing. Nevertheless, there is no reason to rule out either Gaussianity or linearity a priori. Hence, we fit four specifications in total (to each sub-sample). First, a benchmark model with Gaussian marginals and linear dependence—a copula model equivalent to a conventional regression, with linear dependence. Second, maintaining the assumption of linearity for dependence, we consider non-Gaussian marginals, with estimation based on the pseudo data. Third, we consider non-linear dependence, but adopt Gaussian marginals. And, for the final most general specification, we consider non-Gaussian marginals with non-linear dependence.

2.3 Prediction and Evaluation

Prediction involves simulation directly from the copula density on the unit cube for all four specifications. Then, we exploit the appropriate fitted marginal for each specification to produce predictions on the observable scale for output growth. The last step uses the inverse marginal for output growth, $F_1^{-1}(\cdot)$, to generate predictions with the required distribution for the target variable. We deploy the inverse ECDF for the two non-Gaussian specifications and the inverse Normal for the two Gaussian specifications.[1]

Turning to the assessment of predictive accuracy for our four specifications (for each sub-sample of historical data), we examine both relative Root Mean Squared Errors (RMSE) and a relative entropy based measure of probabilistic accuracy. The RMSE approach computes the square root of the average of squared errors from the conditional mean prediction for each specification, with the lowest RMSE preferred. The idea behind the relative entropy approach is to select a specification which on average gives highest probability to the data. A Bayesian interpretation of this Kullback-Leibler Information Criterion (KLIC) approach to model selection is given by Fernandez-Villaverde and Rubio-Ramirez (2004). See, for further details, the discussions in Kullback and Leibler (1951), Kullback (1987), and Roberston, Tallman and Whiteman (2005). As noted by, for example, Kascha and Ravazzolo (2010), under some regularity conditions KLIC optimization is equivalent to minimizing the average logarithmic score (log score) of the densities using the sample data.

3 Data

Given that we look at evidence across a variety of sub-samples and measures of macroeconomic variables, we describe the data considerations in detail. To facilitate comparisons with ABG (2019a), we examine their preferred measure of financial conditions for modern data. In addition, we explore other measures of financial conditions germane for our historical sub-samples, namely the credit spread, the term spread and a measure of stock market volatility. Empirical studies using these alternative measures and historical data include Bernanke (1983), Coe (2002) and Bordo and Haubrich (2008, 2010). We also consider similar measures of economic growth to ABG (2019a), albeit using GNP rather than GDP to give a consistent series across historical sub-samples.

Reflecting the heterogeneity in the scale of economic fluctuations through our 100+ year span of quarterly data, we divide our sample into four sub-samples and fit all four empirical specifications for each era. The first of these, 1875:1 to 1913:4, covers the classical Gold Standard. The second, 1919:1 to 1941:3, begins just after World War I and ends just prior to the U.S. entry into World War II.[2] The third era, 1946:1 to 1971:2, corresponds to the Bretton Woods exchange rate system. The fourth era, 1971:3 to 2017:4, starts with the demise of the Bretton Woods system and includes both the Great Moderation and the 2008 financial crisis. Bordo and Haubrich (2008, 2010) and Schularick and Taylor (2012) use similar era dates. ABG (2019a) consider the most recent (modern) era only.

We follow ABG (2019a) in reporting results for both annualized one and four quarter growth rates of economic growth. We collected our quarterly GNP data from 1875 to 2017 using the Federal Reserve Economic Data (FRED) database and Balke and Gordon (1986). The length of the sub-samples varies by era, with 154, 90, 101 and 185 observations for the one quarter measure, and 148, 87, 98 and 182 for the four quarter measure. We used the real GNP series from Balke and Gordon to construct the growth measures for the first three eras. The FRED database supplied the real GNP series for the modern era. Further details of the sources and construction of all variables are available in the online appendix.

Since the Chicago Fed's National Financial Conditions Index (NCFI) only covers the modern sub-sample from the early 1970s, we explored alternative measures of financial conditions for all eras. These are the credit spread, the term spread and a measure of stock market volatility. For the first era, we used Railway Bond yields from the NBER Macrohistory database, reported by Macaulay (1938), to construct a credit spread. For the remaining three eras, we used the Baa-Aaa spread calculated from FRED data. For term spreads in our first three eras, we followed Balke and Gordon (1986), using Railway Bond Yields and the Moody's Baa rated bonds to construct a long-term corporate bond yield, and subtract from that the 60-90 day New York City Commercial Paper rate. These data are from FRED and Macaulay (1938). We used the difference between the 10-Year Treasury Constant Maturity Rate and the 3-Month Treasury Bill rate to represent the term spread, with both series taken from FRED for the final era. Finally, we used Shiller's (2000) S&P Composite Index to measure stock market volatility for the four sub-samples.[3]

There are differences in scale across eras for output growth, with variation in the unconditional means and standard deviations. For the one quarter growth measure, the first era has the highest unconditional mean at 4.08 and the second era has the highest standard deviation at around 12.5. The second era has the lowest unconditional mean at 2.75. This is similar to the mean for the modern sub-sample, 2.77. The modern era has the lowest standard deviation at 3.22. Therefore, while the sub-sample including the Great Moderation and the Great Recession has seen relative stability in economic growth, mean growth is close to that of the sub-sample that includes the Great Depression. We see the same patterns in the four quarter growth measure.[4]

4 Results

In this section, we analyze the evidence supporting both non-Gaussianity and non-linearity, and assess whether financial conditions have predictive content for economic growth in our various historical sub-samples. Then, we present our conditional output growth predictive densities and illustrate the contribution of financial conditions with some well-known historical financial crises. We begin with an ocular assessment of the fitted marginal distributions for output growth by era. This provides further support for non-Gaussianity in general but highlights the complex and varied forms of departures from Gaussianity across eras .

4.1 Fitted Marginal Distributions

Figure 1 provides eight plots displaying the fitted marginal distributions (plotted as probability density functions, smoothed for illustration) from the ECDFs for output growth for each era.[5] The four rows of plots refer to the respective eras, ordered by start date, with (for example) the modern era shown in the bottom row. We provide results for two economic growth measures, with plots based on the one quarter GNP growth measure in the left column, and the four quarter measure in the right column.

Looking at the top left panel, namely one quarter growth during the Gold Standard era, we see the density exhibits an asymmetric distribution, with negative skew overall, a slightly longer and thinner left tail, and a slightly shorter and fatter right tail. Turning to the modern sub-sample, post-1970, we see a similar pattern in the bottom left panel. ABG (2019a) noted the considerable probability mass to the right of the mode, reflecting the long upswing of the business cycle. Looking at the third row, Bretton Woods, we again see similar features. However, the second era, which includes the Great Depression, has a distinct almost triangular shape indicating considerable probability mass to the left of the mode. Put differently, U.S. output growth was unusually "vulnerable" during the years between the wars, which includes the Great Depression. The plots in the right column represent the marginal distributions for four quarter growth and display a similar pattern. In general, though, the shapes of the densities across eras (rows) and output growth measures (columns) are similar (ignoring the obvious differences in scale) and suggest departures from Gaussianity which are confirmed by the Shapiro-Wilk tests reported in the online appendix.[6]

Overall, we conclude from our ocular assessment that the fitted marginal distributions are typically non-Gaussian but in complex ways that are not easy to reconcile with parametric distributions. This motivates our consideration of non-Gaussian marginal distributions in our subsequent analysis.

4.2 Assessing Predictive Content

Our results are robust across measures of financial conditions. However, to be concise, we only report results for a single measure of financial conditions for each era. For the modern era we use the NFCI to facilitate comparison with ABG (2019a). We report results based on the credit spread measure of financial conditions for the interwar and Bretton Woods sub-samples. Finally, we report results based on the term spread for the Gold Standard era, as our credit spread measure is based on railway bonds only for this era. Table 1 summarizes the in-sample fit of our various specifications using RMSE and the KLIC-based measure of predictive performance. For each subsample we report results with and without our preferred measure of financial conditions. See the online appendix for similar results using alternative measures of financial conditions.[7]

Panel A of Table 1 displays the results for RMSE, and panel B the results for the KLIC-based measure of predictive performance. Within each panel we report results for four specifications. The first is the baseline specification with Gaussian marginals and linear dependence—equivalent to a conventional linear regression. The second is the specification with non-Gaussian marginals and linear dependence. The third is the case with Gaussian marginals and non-linear dependence. And, the fourth is the most general specification, with non-Gaussian marginals and non-linear dependence. For each specification we consider two variants, one without financial conditions and one with a measure of financial conditions. The eight columns in Table 1 are based on our two measures of output growth and four sub-samples. The first four columns refer to one quarter economic growth and the last four refer to the four quarter equivalent. Recall that throughout, we condition on lagged output growth—matching the variables of interest in ABG (2019a).

Looking at panel A of Table 1, reporting the relative RMSE of our various specifications, we see that adding financial conditions adds little to the fit of the benchmark specification. Generally, the gains from including this variable are less than five percent, regardless of the measure of output growth. The one exception is for the first sub-sample and the four quarter output growth measure, where the gain is above ten percent. For the modern data, the gains are around five percent of RMSE relative to the benchmark regardless of the measure of output growth.

Turning to our second specification, we see fairly small gains in predicability from adopting non-Gaussian marginals with linear dependence. Without financial conditions, RMSE is close to the benchmark. And, adding financial conditions rarely makes much difference to predictability over the corresponding cases with Gaussian marginals. For example, when adding financial conditions, the gains are again close to five percent relative to the benchmark for both measures of output growth using modern era data.

In contrast, when we move away from linear dependence, we see much larger predictability gains from adding financial conditions. With Gaussian marginals and non-linear dependence, but ignoring financial conditions, the improvement in RMSE is typically under 10 percent, regardless of sub-sample or measure of output growth. There are substantive gains from including financial conditions for all eras. For example the predictive gain is around 20 percent during the Gold Standard era for the one quarter economic growth measure relative to the benchmark. For Great Depression and Bretton Woods eras, the corresponding gains are above 30 percent for the same measure of output growth. For the four quarter measure, the gains are in the region of 25 to 40 percent with some variation across historical eras. The gain is approximately 15 to 30 percent for both measures of output growth in the modern era. Overall, we conclude that there is strong evidence for the inclusion of financial conditions in all sub-samples, with non-linear dependence and Gaussian marginals. And some evidence that non-linearity matters even if we do not consider financial conditions.

Considering the last specification, we see stronger evidence for non-Gaussianity and non-linear dependence. Across all sub-samples and for all measures of output growth, relative RMSEs are below one, before the addition of financial conditions, with gains in the region of five to 11 percent. Turning to the issue of whether financial conditions matter for this more general specification, we see large predictive gains for all eras. For example, the predictive gain is around 25 percent for the Gold Standard era using the one quarter economic growth measure. For the subsequent eras, the gains are as much as 40 percent; see, for example, the second era (including the Great Depression) where the gain is around 42 percent. Similar gains in predictability from including the financial variables are displayed across all sub-samples for the four quarter measure of output growth. For example, the modern era gain is in the region of 25 to 30 percent relative to the benchmark.

Overall, we have three main findings based on our results presented in panel A of Table 1. First, in terms of ranking the predictability of our various specifications, the non-Gaussian and non-linear model is best, followed by the Gaussian and non-linear specification. Second, there is strong evidence for the inclusion of financial conditions in all sub-samples with our preferred specification. Third, even in the specification with Gaussian marginals and non-linear dependence, including financial conditions matters for predictability, regardless of the sub-sample considered. So much so, that without financial conditions there is only a modest performance gain from moving beyond the conventional linear benchmark. These findings are robust across measures of output growth and measures of financial conditions.

Turning now to the consideration of the KLIC in panel B of Table 1, we draw similar results in terms of probabilistic predictability—of particular relevance for risk considerations. Regardless of the measure of output growth, including financial conditions improves predictability in our preferred non-Gaussian and non-linear specification. As with RMSE, the KLIC results indicate that Gaussian specifications with non-linear dependence rank second overall in terms of predictability with financial conditions included. From a probabilistic perspective, there is a small predictability gain relative to the benchmark for the Gaussian and non-linear specification ignoring financial conditions.

4.3 Conditional Output Growth Densities

Figures 2 and 3 plot the means of the conditional distribution for output growth along with 5^{th} and 95^{th} percentiles over time and the data realizations. In all cases the conditional distributions use our preferred empirical specification with non-Gaussian (ECDF) marginals and non-linear dependence. In these figures we provide results using the same measures of financial conditions as in Table 1, and report similar plots for other measures of financial conditions in the online appendix. In the left column of these figures, we condition only on lagged output; and, for the right column, we condition on both lagged output growth and lagged financial conditions. Each row refers to a specific era. Figure 2 reports plots for the one quarter growth measure and Figure 3 does the same for the four quarter growth measure.

Two features are immediately apparent when looking at these figures. First, with the addition of financial conditions, the conditional mean tracks the data much more closely, consistent with the large drop in RMSE reported in Table 1. This characteristic emerges for realizations of output growth in the lower tail of the unconditional distribution, and also for realizations closer to the historical mean. Second, the difference between the 5^{th} and 95^{th} percentiles is much smaller in the panels on the right—indicating reduced uncertainty when accounting for financial conditions. These features occur regardless of the measures of output growth.

Looking more closely at the bottom rows of Figures 2 and 3, which refer to the modern era, the conditional mean declines around 2008 for both measures of output growth when including financial conditions. Furthermore, there is a coincident drop in the 5^{th} percentile—consistent with the conclusion that financial conditions contributed to increased growth vulnerabilities. In other panels of Figures 2 and 3 we see a similar pattern in the conditional mean and 5^{th} percentile around 1893, 1907 and 1929. These are years in our historical eras which are identified as having financial crises in the Jordá, Schularick and Taylor (2017) Macrohistory Database. Finally in all rows of Figures 2 and 3, we see more variability in the 95^{th} percentile as well as in the 5^{th} percentile with the addition of financial conditions, consistent with financial conditions containing information for upside as well as downside risks to economic growth.

4.4 Selected Financial Crises

ABG (2019a) report conditional probability densities for output growth in the quarter and in the year following the 2008 financial crisis. They find changes consistent with increased vulnerability of economic growth when they add financial conditions. Figure 4 plots the conditional predictive densities for output growth during four well-known examples of financial crises, namely, the second quarter of 1893, the fourth quarter of 1907, the fourth quarter of 1929 and the fourth quarter of 2008. Each row refers to a specific crisis and the panels on the left refer to output growth in the quarter following the financial crisis and the panels on the right refer to output growth in the year following the financial crisis. In each panel, we plot the predictive densities with and without (lagged) financial conditions, for our preferred non-Gaussian and non-linear specification (conditioning on lagged output growth throughout).

The top row of Figure 4 plots the predictive densities for the 1893 crisis, which we assume began in the second quarter of 1893 to coincide with the decline in stock prices and the bank runs in June of that year; see, for example, Friedman and Schwartz (1963). For the one quarter measure of economic growth (left column), the conditional predictive density without financial conditions (in black) displays evidence of negative skew, with a mode slightly below zero. The corresponding predictive density with financial conditions (in red) exhibits bi-modality, with twin peaks in probability at approximately zero and -25 percent economic growth. For the year after the 1893 crisis, the twin peaks occur at approximately -10 and two percent economic growth. Disregarding financial conditions for the one year ahead case results in just a single probability peak at around seven percent economic growth. Financial conditions radically change the assessments of downside risks to economic growth around the 1893 crisis.

The second row of Figure 4 plots the predictive densities for the 1907 crisis following the failure of the Knickerbocker Trust and the subsequent run on New York trust companies in October; see, for example, Frydman et al (2015). The predictive densities are plotted with (red) and without (black) financial conditions for the subsequent quarter (left column) and year (right column). These predictive densities indicate higher probability of negative economic growth with financial conditions for both horizons at around 90 percent (one quarter) and 25 percent (four quarter). As with the 1893 crisis, there is evidence of bi-modality only when accounting for financial conditions.

The third row of Figure 4 plots the predictive densities for output growth following the 1929:4 stock market crash. At both the one quarter and one year ahead horizons, the probability of negative output growth is markedly greater if we condition on financial conditions. For one quarter ahead (left column), the probability of negative growth is nearly 100 percent. For the year ahead (right column) the corresponding probability is around 80 percent. Without financial conditions, the probabilities drop to around 70 percent and 30 percent, respectively. Interestingly, although the four quarter ahead predictive density is bi-modal, the one quarter ahead predictive is unimodal when accounting for financial conditions.

The bottom row of Figure 4 plots predictive densities for the fourth quarter of 2008. We use the same NFCI measure of financial conditions as ABG (2019a).[8] The lower conditional predicted mean and the (previously noted) increase in downside risks for both the quarter and the year following 2008:4 are apparent from the predictive densities with financial conditions (red) and without (black). However, the impact of financial conditions on downside risk is more modest for 2008 than for the earlier crises.

We emphasize the bi-modality evident in five out of the six cases when conditioning on financial conditions for the 1893, 1907 and 1929 crises. This property contributes considerably to the downside risk assessments during historical crises. Ignoring financial conditions results in weaker downside risk and typically unimodal probability density functions. Although financial conditions also influence the assessment of downside risk for the 2008 crisis, the effect is less pronounced and the predictive densities are unimodal. The historical evidence presented here suggests even stronger impacts of financial conditions on downside risk for economic growth than for the most recent crisis.

5 Conclusion

Our analysis of the historical evidence supports the empirical connection between financial conditions and economic growth in the United States, building on earlier work by Adrian, Boyarchenko and Giannone (2019a) and others. Our consideration of a variety of measures of economic growth and financial conditions establishes the robustness of the relationship across sub-samples of quarterly historical data from 1875 to 2017. We find that both non-Gaussianity and non-linearity shape the historical relationship between these important macroeconomic variables.

Given the effectiveness of the NFCI as a predictor for economic growth on modern data documented by Adrian, Boyarchenko and Giannone (2019a), a corresponding index suitable for historical analysis would be helpful for subsequent research. This would permit probabilistic assessments for historical events based on a still wider set of financial conditions measures using non-Gaussian and non-linear specifications.

Notes

[1]Therefore, for the specification non-Gaussian marginals and linear dependence, the implied correlation between observables exhibits (slight) non-linearity as a result of the non-linear transform associated with the inverse ECDF.

[2]We find very similar results when our second era spans 1914:1 to 1945:4 and so includes both world wars. These results are reported in the online appendix.

[3]See http://www.econ.yale.edu/ shiller/data.htm for the most recent data.

[4]A more comprehensive set of summary statistics for all variables is available in the online appendix.

[5]The locally-adaptive univariate KDE developed by Shimazaki and Shinomoto (2010) was used for all results described below.

[6]In the online appendix we also report marginal distributions and summary statistics for our various measures of financial conditions across our four eras. In general these are also hard to reconcile with Gaussian distributions.

[7]In the online appendix, we explore specifications era-specific marginal distributions but common dependence across eras. Although these specifications fit less well, the results are economically similar to those reported in the main text. We also report recursive results to demonstrate the consistent ranking of our preferred empirical copula specification and the benchmark in the online appendix.

[8]ABG (2019a) report similar results using a term spread, a credit spread and a measure of stock market volatility. We report qualitatively similar results in the online appendix, along with results using alternative measures of financial conditions for other eras.

References

Adrian, Tobias, Nina Boyarchenko, and Domenico Giannone (2019a) “Vulnerable Growth” *American Economic Review* 109(4) pp1263-1289.

Adrian, Tobias, Nina Boyarchenko, and Domenico Giannone (2019b) “Multimodality in Macro-Financial Dynamics” unpublished manuscript October.

Adrian, Tobias, Federico Grinberg, Nellie Liang, and Sheheryar Malik (2018) “The Term Structure of Growth-at-Risk” *IMF Working Paper* WP/18/180.

Amengual, Dante, Enrique Sentana and Zhanyuan Tian (2019) “Gaussian Rank Correlation and Regression” unpublished manuscript.

Balke, Nathan S. and Robert J. Gordon (1986) “Appendix B: Historical Data” in Robert J. Gordon ed. *The American Business Cycle: Continuity and Change* University of Chicago Press, Chicago.

Bernanke, Ben S. (1983) “Nonmonetary Effects of the Financial Crisis in the Propagation of the Great Depression” *American Economic Review* 73(3) pp257-276.

Bernard, Carole and Claudia Czado (2015) “Conditional Quantiles and Tail Dependence” *Journal of Multivariate Analysis* 138 pp104-126.

Bordo, Michael D. and Joseph G. Haubrich (2008) “The Yield Curve and a Predictor of Growth: Long-run Evidence, 1875 - 1997” *Review of Economics and Statistics* 90(1) pp182-185.

Bordo, Michael D. and Joseph G. Haubrich (2010) “Credit Crises, Money and Contractions: An Historical View” *Journal of Monetary Economics* 57(1) pp1-18.

Carriero, Andrea, Todd E. Clark and Massimiliano Marcellino (2019) "Capturing Macroeconomic Tail Risks with Bayesian Vector Autoregressions" unpublished manuscript.

Charpentier, Arthur, Jean-David Fermanian, and Ollivier Scaillet (2007) *The Estimation of Copulas: Theory and Practice, Copulas: from Theory to Applications in Finance.*

Chavleishvili, Sulkhan and Simone Manganelli (2019) "Forecasting and Stress Testing with Quantile Vector Autoregression" unpublished manuscript.

Chen, Song X. (1999) "Beta Kernel Estimators for Density Functions" *Computational Statistics and Data Analysis* 31(2) pp131-145.

Coe, Patrick J. (2002) "Financial Crisis and the Great Depression: A Regime-Switching Approach" *Journal of Money, Credit and Banking* 34(1) pp76-93.

Deheuvels, Paul (1979) "La Fonction de Dépendence Empirique et Ses Propriétés. Un Test non Paramétrique d'Indépendence" *Bulletin Royal Belge de l'Académie des Sciences* 65 pp274-292.

Deheuvels, Paul (1981) "An Asymptotic Decomposition for Multivariate Distribution-free Tests of Independence" *Journal of Multivariate Analysis* 11(1) pp102-113.

DiNardo, John and Justing L. Tobias (2001) "Nonparametric Density and Regression Estimation" *Journal of Economic Perspectives* 15(4) pp11-28.

Diebold, Francis X. and Roberto S. Mariano (1995) "Comparing Predictive Accuracy" *Journal of Business and Economic Statistics* 13(3) pp. 253-63.

Fernández-Villaverde, Jesús and Juan F. Rubio-Ramirez (2004), "Comparing Dynamic Equilibrium Models to Data: a Bayesian Approach" *Journal of Econometrics* 123(1) pp153-187.

Ferrara, Laurent, M. Mogliani and J-G. Sahuc "Real-time High-frequency Monitoring of Growth-at-Risk" unpublished manuscript.

Friedman, Milton and Anna J. Schwartz (1963) *A Monetary History of the United States* Princeton: Princeton University Press.

Frydman, Carola, Eric Hilt and Lily Y. Zhou (2015) "Economic Effects of Runs on Early "Shadow Banks": Trust Companies and the Impact of the Panic of 1907" *Journal of Political Economy* 123(4) pp902-940.

Harvey, David, Stephen Leybourne, and Paul Newbold (1997) "Testing the Equality of Prediction Mean Squared Errors" *International Journal of Forecasting* 13(2) pp281-291.

Joe, Harry (2006) "Generating Random Correlation Matrices Based on Partial Correlations" *Journal of Multivariate Analysis* 97(10) pp2177-2189.

Jordà, Òscar, Moritz Schularick, and Alan M. Taylor (2017) "Macrofinancial History and the New Business Cycle Facts" in *NBER Macroeconomics Annual 2016 Volume 31*, edited by Martin Eichenbaum and Jonathan A. Parker, Chicago: University of Chicago Press.

Karagedikli, Ozer, Shaun P. Vahey and Elizabeth C. Wakerly (2019) "Improved Methods for Combining Point Forecasts for an Asymmetrically Distributed Variable" CAMA Working Paper No. 15/2019.

Karra, Kiran (2018) "Modeling and Analysis of Non-Linear Dependencies using Copulas, with Applications to Machine Learning" PhD thesis, Virginia Polytechnic Institute and State University.

Kascha, Christian and Francesco Ravazzolo (2010) "Combining Inflation Density Forecasts" *Journal of Forecasting* 29(1-2) pp231-250.

Kullback, S. (1987) "Letter to the Editor: The Kullback-Leibler Distance" *The American Statistician* 41(4) pp340-341.

Kullback, S. and R.A. Leibler (1951) "On Information and Sufficiency" *Annals of Mathematical Statistics* 22(1) pp79-86.

Li, Qi and Jeffrey S. Racine (2006) *Nonparametric Econometrics: Theory and Practice*, Princeton University Press, Princeton.

Loaiza-Maya, R. and M.S. Smith (2020) "Real-Time Macroeconomic Forecasting with a Heteroskedastic Inversion Copula" *Journal of Business and Economic Statistics* 38(2) pp470-486.

Loria, Francesca, Christian Matthes, and Donghai Zhang (2019) "Assessing Macroeconomic Tail Risk" unpublished manuscript.

Macaulay, Frederick R. (1938) *The Movement of Interest Rates, Bond Yields and Stock Prices in the United States Since 1856* NBER.

Nelsen, Roger B. (2006) *An Introduction to Copulas*, 2nd ed., New York: NY: Springer.

Robertson, John. C., Ellis W. Tallman and Charles H. Whiteman (2005) "Forecasting Using Relative Entropy" *Journal of Money, Credit and Banking* 37(3) pp383-401.

Schularick, Moritz and Alan M. Taylor (2012) "Credit Booms Gone Bust: Monetary Policy, Leverage Cycles and Financial Crises, 1870 - 2008" *American Economic Review* 102(2) pp1029-1061.

Scott, David. W. (1992) *Multivariate Density Estimation: Theory, Practice, and Visualization*, John Wiley and Sons, New York.

Shiller, Robert J. (2000) *Irrational Exhuberence*, Princeton University Press, Princeton.

Shimazaki, Hideaki and Shigeru Shinomoto (2010) “Kernel Bandwidth Optimization in Spike Rate Estimation” *Journal of Computational Neuroscience* 29(1-2) pp171-182.

Silverman, Bernard. W. (1986) *Density Estimation for Statistics and Data Analysis*, Chapman and Hall, New York.

Smith, Michael S. (2015) “Copula Modelling of Dependence in Multivariate Time Series” *International Journal of Forecasting* 31(3) pp815-833.

Smith, Michael S. and Shaun P. Vahey (2016) “Asymmetric Forecast Densities for U.S. Macroeconomic Variables from a Gaussian Copula Model of Cross-Sectional and Serial Dependence” *Journal of Business and Economic Statistics* 34(3) pp416-34.

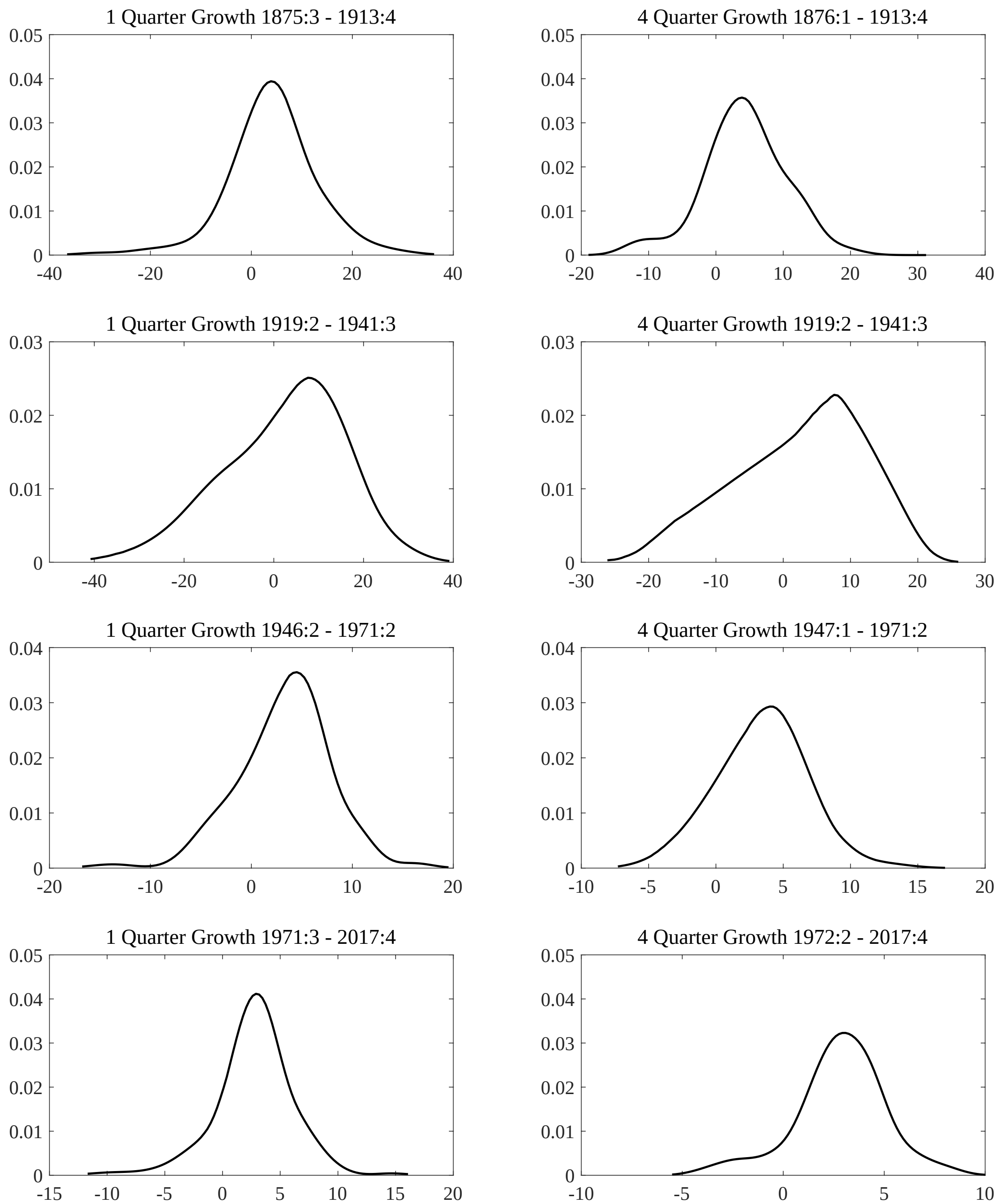
Velásquez-Giraldo, Mateo, Gustavo Canavire Bacarreza, Kim P. Huynh and David T. Jacho-Chavez (2018) “Flexible Estimation of Demand Systems: A Copula Approach” *Journal of Applied Econometrics* 33(7) pp1109-1116.

Table 1: RMSE and KLIC Based Measures of In-Sample Fit

	1 Quarter Growth				4 Quarter Growth			
	1875:3-1913:4	1919:2-1941:3	1946:2-1971:2	1971:4-2017:4	1877:1-1913:4	1920:1-1941:3	1947:1-1971:2	1972:3-2017:4
(A) ROOT MEAN SQUARED ERROR								
Gaussian Marginals and Linear Dependence								
No Financial Conditions	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Financial Conditions	0.947*	1.002	1.001	0.956*	0.884***	0.988	0.999	0.953**
Non-Gaussian Marginals and Linear Dependence								
No Financial Conditions	0.999	1.003	1.005	1.005	1.002	1.005	1.000	0.993
Financial Conditions	0.945*	1.004	1.009	0.974	0.887***	0.991	0.998	0.955**
Gaussian Marginals and Non-linear Dependence								
No Financial Conditions	0.982	0.928**	0.942**	0.988	0.963*	0.943*	0.875***	0.939***
Financial Conditions	0.784***	0.669***	0.693***	0.836***	0.628***	0.724***	0.678***	0.691***
Non-Gaussian Marginals and Non-linear Dependence								
No Financial Conditions	0.969**	0.919**	0.955**	0.990	0.954**	0.950**	0.889***	0.936***
Financial Conditions	0.757***	0.584***	0.665***	0.752***	0.619***	0.574***	0.577***	0.681***
(B) KLIC BASED MEASURE								
Gaussian Marginals and Linear Dependence								
No Financial Conditions	1.000	1.000***	1.000***	1.000	1.000	1.000***	1.000***	1.000
Financial Conditions	0.983**	1.000	0.999	0.987**	0.969***	0.998	1.000	0.987*
Non-Gaussian Marginals and Linear Dependence								
No Financial Conditions	0.980**	0.995	0.991	0.982**	0.991**	0.993	0.998	0.983***
Financial Conditions	0.967**	0.995	0.990	0.975***	0.961***	0.990	0.998	0.976***
Gaussian Marginals and Non-linear Dependence								
No Financial Conditions	0.943***	0.942***	0.945***	0.993	0.943***	0.958***	0.957**	0.951***
Financial Conditions	0.804***	0.830***	0.823***	0.915**	0.809***	0.869***	0.843***	0.860***
Non-Gaussian Marginals and Non-linear Dependence								
No Financial Conditions	0.935***	0.945***	0.949***	0.951***	0.939***	0.953***	0.934***	0.936***
Financial Conditions	0.795***	0.809***	0.798***	0.800***	0.802***	0.809***	0.785***	0.804***

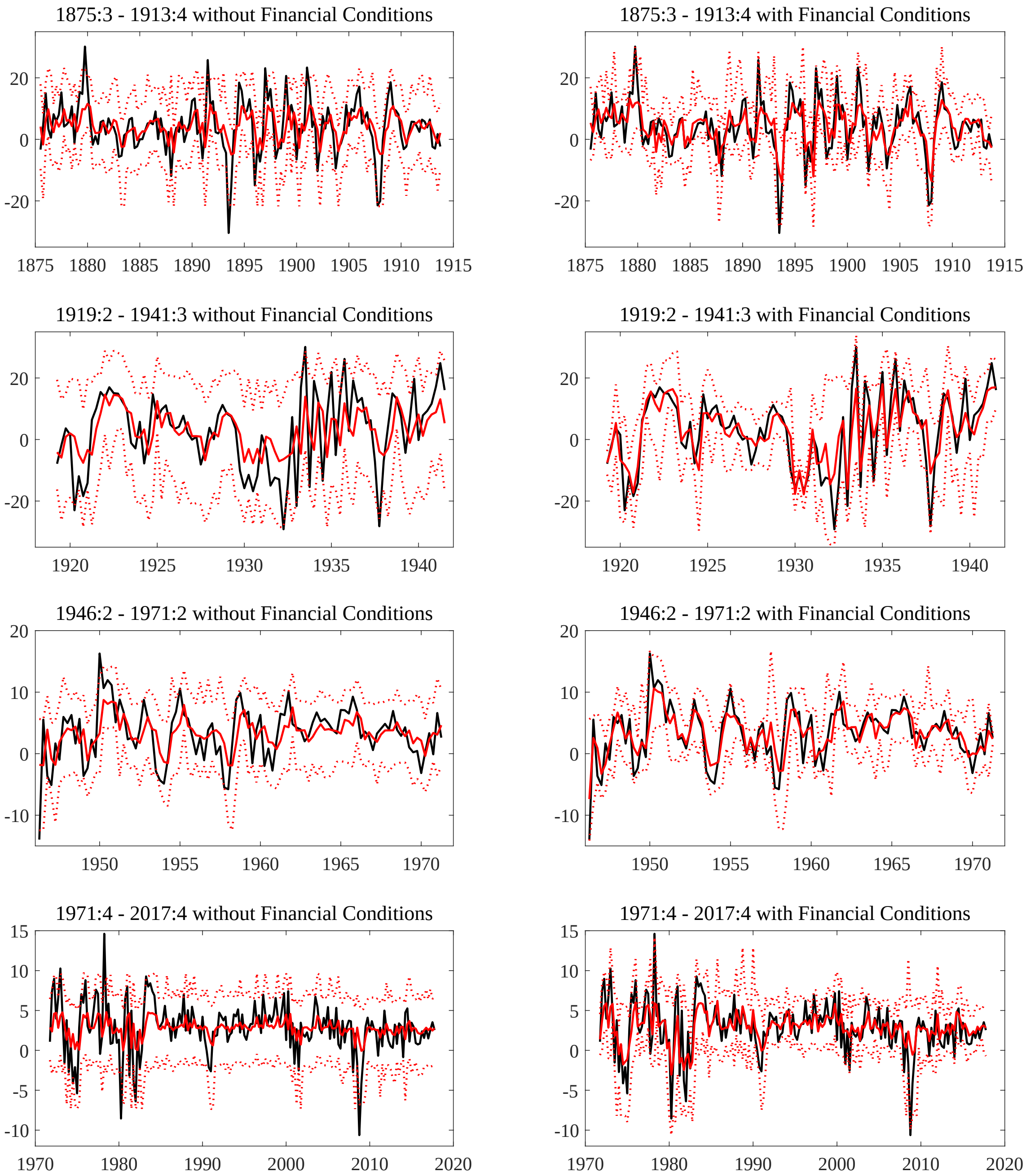
Notes: (1) Root Mean Squared Errors and KLIC based measures of fit are reported relative to the benchmark model with Gaussian marginals, linear dependence and no financial conditions. (2) As a rough guide to statistical significance for RMSE, p-values of a Harvey, Leybourne, and Newbold (1997) small-sample adjustment of the two-sided Diebold and Mariano (1995) test are denoted by $*$ (< 0.10), $**$ (< 0.05) and $***$ (< 0.01). Similarly, as a rough guide to statistical significance for the KLIC based measure, p-values of a two-sided Diebold-Mariano (1995) type test for the log score are denoted by $*$ (< 0.10), $**$ (< 0.05) and $***$ (< 0.01). (3) Financial conditions are represented by the term spread for the first sub-sample, the credit spread for the second and third sub-samples and the Chicago Fed NFCI for the last sub-sample.

Figure 1: Fitted Marginal Distributions



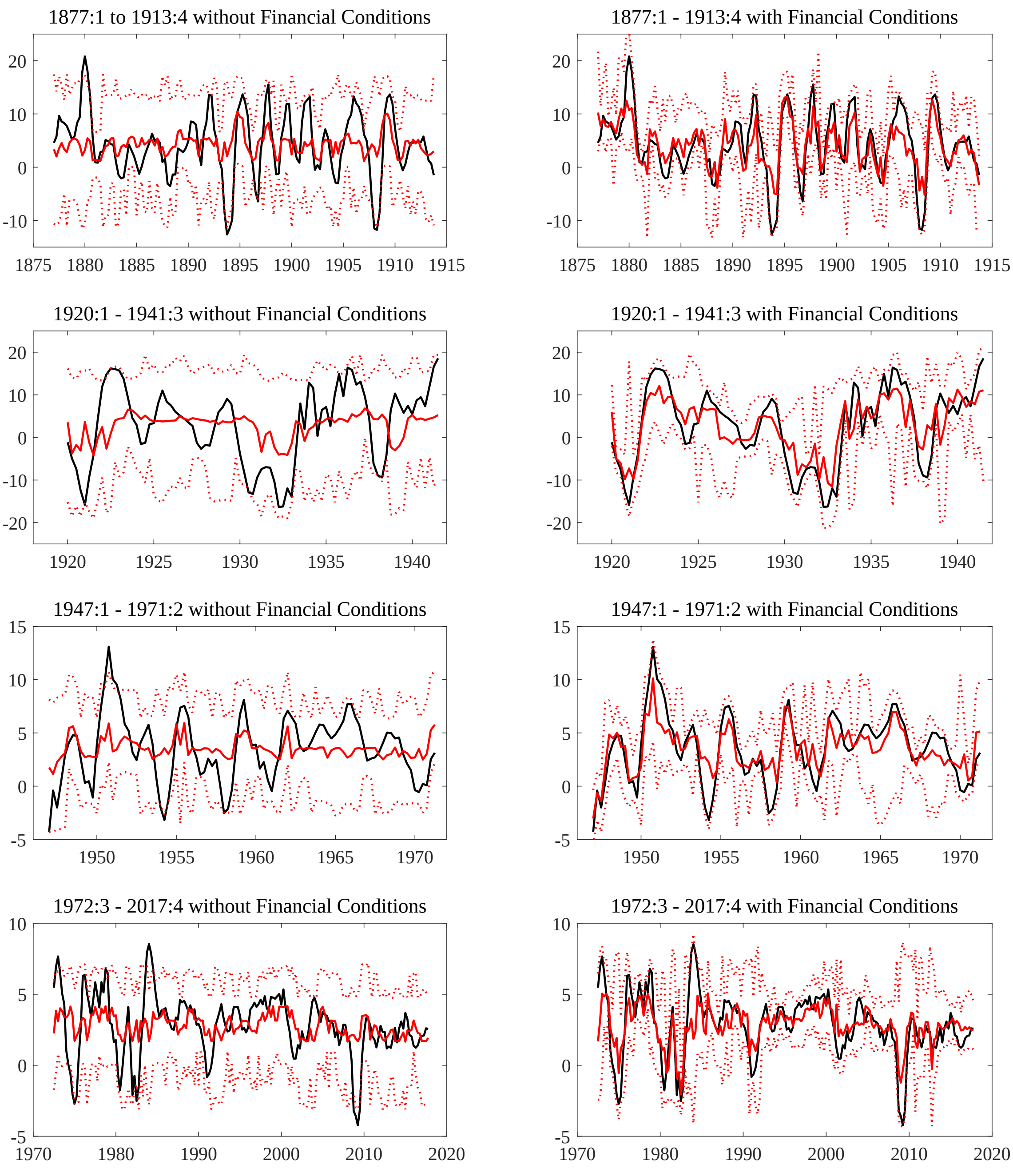
Note: The panels depict the marginal distribution for output growth in each era, fitted using the SSV method of Shimazaki and Shinomoto (2010) and plotted as PDFs.

Figure 2: Conditional Densities for One Quarter Growth



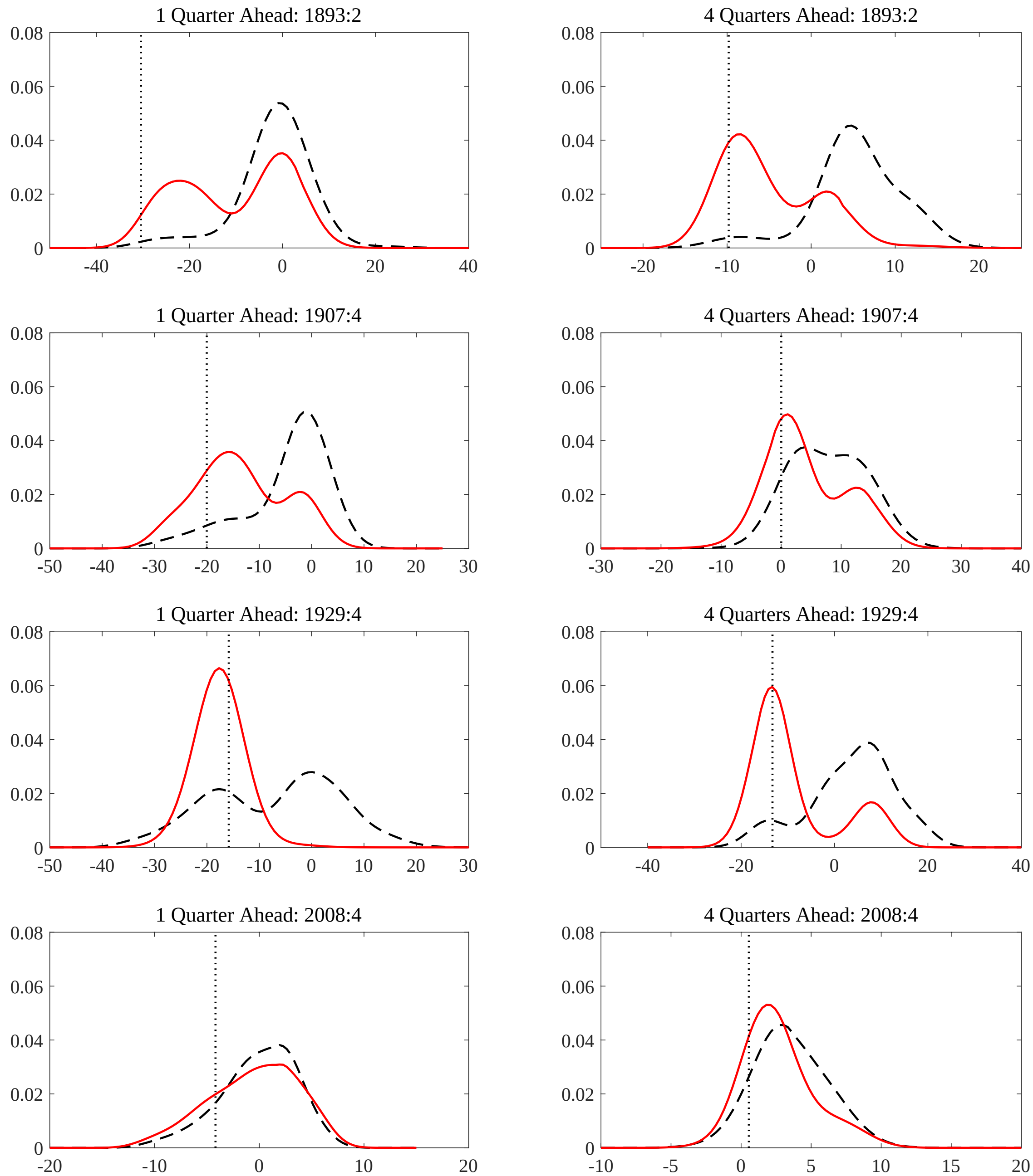
Notes (1): In each panel, the solid line black line depicts the realizations and the solid red line depicts the mean of the conditional density for output growth based on a model with non-Gaussian marginals and non-linear dependence. The dotted red lines depict the 5^{th} and 95^{th} percentiles. (2) Financial conditions are represented by the term spread for the first sub-sample, the credit spread for the second and third sub-samples and the Chicago Fed NFCI for the last sub-sample.

Figure 3: Conditional Densities for Four Quarter Growth



Note: See note to Figure 2.

Figure 4: Predictive Densities For Selected Financial Crises



Notes: (1) The dashed black line depicts the conditional density based on the specification with non-Gaussian marginals and non-linear dependence, without financial conditions. The solid red line depicts the equivalent density accounting for financial conditions. (2) The measures of financial conditions are the term spread (first two rows), the credit spread (third row) and the NFCI (fourth row). (3) The left (right) panels display the densities for output growth in the subsequent quarter (year).